

6. The function  $g$  has derivatives of all orders for all real numbers. The Maclaurin series for  $g$  is given by

$$g(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2e^n + 3} \text{ on its interval of convergence.}$$

- (a) State the conditions necessary to use the integral test to determine convergence of the series  $\sum_{n=0}^{\infty} \frac{1}{e^n}$ .

Use the integral test to show that  $\sum_{n=0}^{\infty} \frac{1}{e^n}$  converges.

- (b) Use the limit comparison test with the series  $\sum_{n=0}^{\infty} \frac{1}{e^n}$  to show that the series  $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$

converges absolutely.

- (c) Determine the radius of convergence of the Maclaurin series for  $g$ .

- (d) The first two terms of the series  $g(1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2e^n + 3}$  are used to approximate  $g(1)$ . Use the alternating

series error bound to determine an upper bound on the error of the approximation.

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